Transmission Capacity Expansion: An Improved Transport Model

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***Abstract*—This paper presents an improved Transport Model called Piecewise Linear Transport Model (PLT). It models physical load flows with a precision comparable to that of the DC model while allowing capacity expansion problems to be solved as Linear Programs (LPs) as is the case for the conventional Transport Model. In addition, case studies are conducted to compare the model with conventional load flow models.**

NOMENCLATURE

$\mathcal{R}$	set of regions
$\mathcal{L}$	set of lines
$o(l)$	origin of line $l \in \mathcal{L}$
$d(l)$	destination of line $l \in \mathcal{L}$
f_l	flow on line $l \in \mathcal{L}$
p_r	power generated in region $r \in \mathcal{R}$
μ_l	loss variable for line $l \in \mathcal{L}$
θ_v	phase angle at vertex v
κ_r	production cost function for region $r \in \mathcal{R}$
κ_l	capacity cost function for line $l \in \mathcal{L}$
D_r	demand in region $r \in \mathcal{R}$
$\mathcal{P}_r$	feasible region for variable p_r
$\mathcal{F}_l$	feasible region for variable f_l
b_l	susceptance of line $l \in \mathcal{L}$
g_l	conductance of line $l \in \mathcal{L}$
η_l	transmission efficiency of line $l \in \mathcal{L}$
γ_l	length- and capacity-invariant parameter of line $l \in \mathcal{L}$
λ_l	length of line $l \in \mathcal{L}$
c_l	capacity of line $l \in \mathcal{L}$
$\mu_l^c(f)$	losses for flow f on line $l \in \mathcal{L}$ with capacity set to c
α_l, β_l	coeff's of linear pieces to approximate losses on line $l \in \mathcal{L}$

I. INTRODUCTION AND PROBLEM DESCRIPTION

Energy systems models with capacity expansion optimization need to cover long time periods in high temporal resolution to take into account fluctuations in renewable electricity generation and storage. The transmission grid plays an important role for investment decisions and thus a sufficiently detailed representation is desirable.

Transmission Capacity Expansion Planning (TEP) has been an important problem in energy systems optimization for the last decades and a vast amount of scientific literature on the issue exists. However, all existing models face one of two serious drawbacks: Some use a very simple network model whose accuracy is rather poor, which limits the meaningfulness of results. Others employ a more sophisticated model which, in turn, allows only to compare a series of predefined expansion options.

In a long term holistic optimization framework, however, both options are questionable: Networks may not be assumed to remain close to some operating point defined now, which makes a precise model of the physical laws governing electricity flows desirable. And the further the entire system may stray from its current structure, the less adequate is seems to limit network expansion to a set of options fixed *a priori*, especially as this set needs to be rather small in order to keep solution times for the mixed-integer problems within acceptable bounds.

We present a new approach to overcome this dichotomy: In contrast to the second class of models mentioned above, our network model does not require a limitation of capacity expansion options to a small set from which to choose. At the same time it represents physical properties of electricity flows to a level of detail far superior to those simple models of the first class mentioned above. For a detailed mathematical derivation and treatment of the model see [1].

We consider the following simplified economic dispatch (ED) problem:

$$\begin{aligned} \min \; & \sum_{r \in \mathcal{R}} \kappa_r(p_r) \\ \text{s.t. } & p_r + \sum_{\substack{l \in \mathcal{L} \\ d(l)=r}} f_l - \sum_{\substack{l \in \mathcal{L} \\ o(l)=r}} f_l = D_r && \forall\, r \in \mathcal{R} \\ & p_r \in \mathcal{P}_r && \forall\, r \in \mathcal{R} \\ & f_l \in \mathcal{F}_l && \forall\, l \in \mathcal{L} \end{aligned} \tag{ED}$$

This problem statement should be complemented by a unit dispatch/unit commitment model specifying the sets $\mathcal{P}_r$ as well as the cost functions κ_r, and a network model specifying the sets $\mathcal{F}_l$. In this paper, we are primarily concerned with the network model.

A second problem that we will consider is the transmission

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978-1-4799-2984-9/13/$31.00 ©2013 IEEE. This paper was published as P. Ahlhaus and P. Stursberg, “Transmission capacity expansion: an improved transport model”, in *Innovative Smart Grid Technologies Europe (ISGT EUROPE), 2013 4th IEEE/PES*, IEEE, 2013, pp. 1–5, URL: http://dx.doi.org/10.1109/ISGTEurope.2013.6695322

capacity expansion problem:

$$\begin{aligned}\min\ & \sum_{r\in\mathcal{R}} \kappa_r(p_r) + \sum_{l\in\mathcal{L}} \kappa_l(c_l) \\ \text{s.t. } & p_r + \sum_{\substack{l\in\mathcal{L}\\ d(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ o(l)=r}} f_l = D_r && \forall\ r\in\mathcal{R} \\ & p_r \in \mathcal{P}_r && \forall\ r\in\mathcal{R} \\ & f_l \in \mathcal{F}_l(c_l) && \forall\ l\in\mathcal{L}\end{aligned} \tag{TEP}$$

In this problem, the feasible sets for the variables f_l vary with the choice of a variable c_l (the line's capacity). The capacity of a line may thus be increased, thereby allowing for more electricity to be transmitted along that line, but this incurs some additional cost in the objective function.

A. Common Network Models

The most widely-used approaches for network models in the above context are the Transport Model and the DC Model:

1) Transport Model: In a Transport Model (used e. g. in [3] and [4]), power flows are restricted by line capacities alone. This model is computationally very advantageous, also for the capacity expansion problem, but its precision in modeling physical load flows is relatively poor.

2) DC Model: In a DC Model (used e. g. in [5] and thoroughly investigated in [6]), line flows on a line are governed by the load flow equation

$$f_l = b_l \cdot (\theta_{d(l)} - \theta_{o(l)}), \tag{I.1}$$

Results obtained from this model match physical observations much better than those from the Transport Model. If, however, capacity expansion problems are to be solved using this model, computationally disadvantageous techniques like Mixed-Integer Programming or Quadratic Programming (see e. g. [7]) need to be employed.

B. Transmission Losses

As part of the underlying simplifications in both the DC and the Transport Model, ohmic losses are neglected. In applications, this flaw is compensated for in different ways:

The simplest idea is to include losses into demand time series (e. g. [8]). This can, for example, be achieved by scaling demand by an appropriately chosen factor > 1 to account for the additional power that needs to be produced in order to compensate transmission losses. A different approach is to measure or estimate losses of each line l in a certain operating point that is deemed likely to occur. This constant amount of lost power μ_l can then be added to to demand in both incident vertices (see plot (1) in Figure 1):

$$p_r + \sum_{\substack{l\in\mathcal{L}\\ d(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ o(l)=r}} f_l = D_r + \sum_{\substack{l\in\mathcal{L}\\ r\in\{d(l),o(l)\}}} \frac{\mu_l}{2} \qquad \forall\ r\in\mathcal{R} \tag{I.2}$$

If a large time horizon is modeled, however, the assumption that the network will always be close to an operating point defined *a priori* is hard to justify. Instead, the model should compute losses that increase with the load on a line and, in particular, are 0 if the line is not used. Here, the easiest approach is to define losses as a constant fraction of the power transmitted on a line (e. g. [3], [4]). This approach is depicted as plot (2) in Figure 1. As in the previous approach, this fraction must be obtained by measuring or estimating the properties of a line in some operating point. The resulting losses can easily be incorporated into a linear optimization model as follows: Denote by $(1-\eta_l)$ the fraction of power that is lost during transmission. We can now allocate the amount of lost power μ_l to the incident vertices as in (I.2):

$$\begin{aligned} &\left.\begin{aligned}\mu_l &\geq (1-\eta_l) f_l \\ \mu_l &\geq -(1-\eta_l) f_l\end{aligned}\right\} \forall\ l\in\mathcal{L} \\ & p_r + \sum_{\substack{l\in\mathcal{L}\\ d(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ o(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ r\in\{d(l),o(l)\}}} \frac{\mu_l}{2} = D_r\ \forall\ r\in\mathcal{R}\end{aligned} \tag{I.3}$$

Although more accurate, the above approach still relies on estimating line properties in some operating point which limits its precision if the system state diverges far from the state used to make these initial estimates.

This reliance may further be reduced by using higher-order polynomials to approximate line losses. Namely, [9] use a quadratic function to approximate losses by

$$\mu_l = \frac{g_l}{b_l^2} f^2. \tag{I.4}$$

This approach (which has previously been published by [7], where losses are expressed as a quadratic function of θ rather than f) is shown as plot (3) in Figure 1. The authors then approximate this quadratic function by a convex piecewise linear function, which can be modeled as an LP as follows (using N segments for each line):

$$\begin{aligned} &\left.\begin{aligned}\mu_l &\geq \alpha_l^1 f_l - \beta_l^1 \\ &\vdots \\ \mu_l &\geq \alpha_l^N f_l - \beta_l^N \\ \mu_l &\geq -\alpha_l^1 f_l - \beta_l^1 \\ &\vdots \\ \mu_l &\geq -\alpha_l^N f_l - \beta_l^N\end{aligned}\right\} \forall\ l\in\mathcal{L} \\ & p_r + \sum_{\substack{l\in\mathcal{L}\\ d(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ o(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ r\in\{d(l),o(l)\}}} \frac{\mu_l}{2} = D_r\ \forall\ r\in\mathcal{R}\end{aligned} \tag{I.5}$$

Note that, theoretically, the "exact" AC loss function could also be approximated by a piecewise linear function and thus included in an LP model. We will, however, stick to quadratic functions, as they present some useful properties shown in the next section.

II. PLT Model

A. Economic Dispatch

Rather than adding the piecewise linear approximation of transmission losses on top of a DC Network Model as in [7]

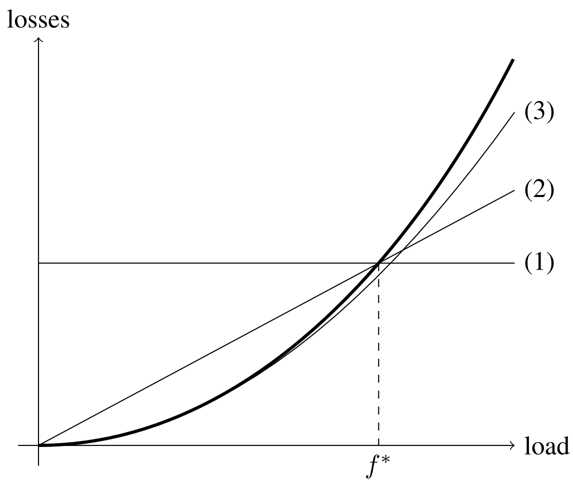


Figure 1. Different models of transmission losses, the thick plot represents the correct physical losses, f^* denotes the operating point used to estimate the parameters for (1) and (2).

Model 1 economic dispatch (ED) problem using PLT network model

$$\min \sum_{r\in\mathcal{R}} \kappa_r(p_r)$$

$$\text{s.t.}\quad \left.\begin{array}{l} p_r + \sum\limits_{\substack{l\in\mathcal{L}\\ d(l)=r}} f_l - \sum\limits_{\substack{l\in\mathcal{L}\\ o(l)=r}} f_l - \sum\limits_{\substack{l\in\mathcal{L}\\ r\in\{d(l),o(l)\}}} \frac{\mu_l}{2} = D_r \\ p_r \in \mathcal{P}_r \end{array}\right\} \quad \forall\ r\in\mathcal{R}$$

$$\left.\begin{array}{c} \mu_l \geq \alpha_l^1 f_l - \beta_l^1 \\ \vdots \\ \mu_l \geq \alpha_l^N f_l - \beta_l^N \\ \mu_l \geq -\alpha_l^1 f_l - \beta_l^1 \\ \vdots \\ \mu_l \geq -\alpha_l^N f_l - \beta_l^N \\ f_l \leq f_l^{\max} \end{array}\right\} \quad \forall\ l\in\mathcal{L}$$

and [9], however, we exploit a property of this approximation that allows us to ignore the DC load flow equations altogether and thus to obtain a Transport-like Network Model which nonetheless models load flows with a precision comparable to that of the DC load flow model.

The PLT Model combines the approach of approximating losses using piecewise-linear functions from [9] with an alternative characterization of the DC power flow based on transmission losses that is used e. g. in [10] but to our knowledge has not been used in recent years. It is motivated by the following theorem. Consider the following optimization problem in f:

$$\begin{aligned} &\min \sum_{l\in\mathcal{L}} \frac{f_l^2}{b_l} \\ &\text{s.t. } D_r - p_r = \sum_{\substack{l\in\mathcal{L}\\ r=d(l)}} f_l - \sum_{\substack{l\in\mathcal{L}\\ r=o(l)}} f_l \qquad \forall\ r\in\mathcal{R} \\ &\qquad f\in\mathbb{R}^m \end{aligned} \tag{T}$$

Then, the following theorem holds:

Theorem II.1. *Suppose, that the problem* (T) *is feasible. Then, the optimal solution f^* of* (T) *fulfills the DC load flow equation*

$$f_l = b_l \cdot (\theta_{d(l)} - \theta_{o(l)}) \quad \forall\ l\in\mathcal{L}.$$

Proof. (T) is a convex optimization problem and we can apply the first-order-condition for optimality: The gradient of the objective function has to be in the linear span of the constraints' normal vectors. We can thus find multipliers θ_r for each region r that fulfill the above equations. □

How does the result from theorem II.1 now relate to the problems (ED) and (TEP) cited above? While the objective functions are certainly different, it seems reasonable to assume that, if a model of quadratic network losses as given in (I.4) is included into the optimization problem, then *in an optimal solution* to (ED), the network flows are not too far from minimizing the sum of losses when production variables p_r are fixed to their optimal solution values. The reason for this is that any additionally incurred losses would have to be balanced by increased production in some vertex, thus leading to additional cost in the objective function of (ED).

In other words, by including quadratic transmission losses – or a piecewise linear approximation thereof as in (I.5) – into the model, we *already* obtain a model that yields close-to-DC network flows without any additional constraints. This argument will be further substantiated by a number of case studies in section III.

The complete PLT version of the problem (ED) is shown in model listing 1.

B. Transmission Capacity Expansion

We now establish some additional useful properties of the above model that allow us to use it to solve transmission capacity expansion problems, as well. Here, it is useful to reconsider the quadratic approximation of transmission losses, this time explicitly taking into account line capacities:

A second-order Taylor approximation of losses μ_l^c on a line l of capacity c_l as a function of transmitted power f_l yields

$$\mu_l^c(f_l) = \frac{\gamma_l}{\lambda_l}\frac{f_l^2}{c_l}. \tag{II.1}$$

In order to obtain an LP model, the (convex) function μ_l^c can be approximated on the interval $[0,c]$ by a constant number N of linear pieces intersecting μ_l^c in $f_l = 0, \frac{c}{N}, \frac{2c}{N}, \ldots, \frac{Nc}{N}$ (for the interval $[-c,0]$, the same pieces with inverted slope may be used). We obtain the following expression for the i-th linear piece:

$$\mu_l \geq \frac{\gamma_l}{\lambda_l}\cdot\left[\frac{2i+1}{N}f - \frac{i^2+i}{N^2}c\right] \tag{II.2}$$

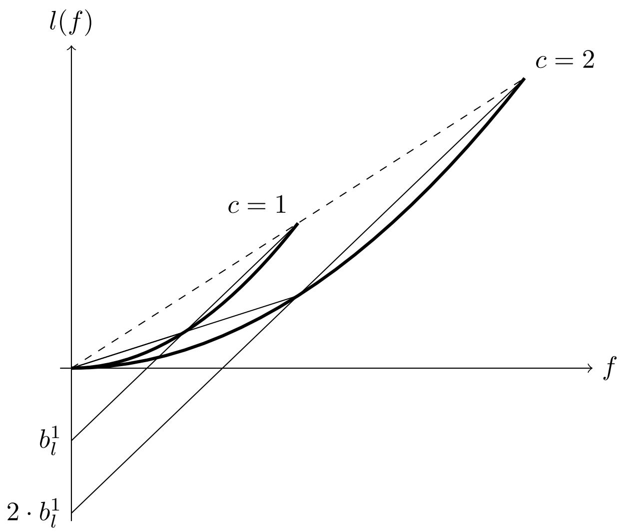


Figure 2. Quadratic loss function for two different values of c together with the respective piecewise linear approximation. Note that the slopes of both pieces remain unchanged, while the intercepts change linearly.

Note, that these inequalities are linear *both in* f *and in* c. We may thus formulate a transmission capacity expansion problem analogous to Model 1, the resulting model is listed as Model2.

It is important to mention that an important advantage of this approach is, as mentioned in [9], that it does not use at all the angle difference between two nodes (which, in contrast to the flow f may be non-zero for non-existing lines). We thus circumvent the computationally disadvantageous *bigM* approaches which are commonly used to make DC load flow equations unbinding for non-existing lines.

Furthermore, it should be noted that it is not necessary to partition the interval $[0, c]$ into equidistant pieces as above. Although it can be shown that the above choice is optimal in terms of the absolute error between the function l and its approximation, it might not be optimal in the context of achieving a network flow closest possible to the DC flow. Here, a dynamic approach as in [11] might prove useful.

III. Results

In the following section the load flow models mentioned above are applied to two different scenarios to demonstrate the accuracy and performance of the introduced PLT model compared to the existing DC and Transport (TR) load flow models. The scenarios used are:

- EU: 83 nodes, 153 lines, 12 timesteps
- DENA: 30 nodes, 52 lines, 100 timesteps

Note that due to the different size and structure of the scenarios, comparing computation times between them is not meaningful. Both test scenarios are solved for both the Economic Dispatch (ED) and Transmission Capacity Expansion (TEP) problem. The same input data are processed with all three different load flow models. Note that, for the test cases,

Model 2 transmission capacity expansion problem (TEP) using PLT network model

$$
\begin{aligned}
\min \quad & \sum_{r\in\mathcal{R}} \kappa_r(p_r) + \sum_{l\in\mathcal{L}} \kappa_l(c_l) \\
\text{s.t.} \quad & \left.\begin{aligned} p_r + \sum_{\substack{l\in\mathcal{L}\\ d(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ o(l)=r}} f_l - \sum_{\substack{l\in\mathcal{L}\\ r\in\{d(l),o(l)\}}} \frac{\mu_l}{2} = D_r \\ p_r \in \mathcal{P}_r \end{aligned}\right\} \quad \forall\, r \in \mathcal{R} \\
& \left.\begin{aligned} \mu_l &\geq \alpha_l^1 f_l - \beta_l^1 c_l \\ &\vdots \\ \mu_l &\geq \alpha_l^N f_l - \beta_l^N c_l \\ \mu_l &\geq -\alpha_l^1 f_l - \beta_l^1 c_l \\ &\vdots \\ \mu_l &\geq -\alpha_l^N f_l - \beta_l^N c_l \\ f_l &\leq c_l \end{aligned}\right\} \quad \forall\, l \in \mathcal{L}
\end{aligned}
$$

		Economic Dispatch DENA	EU
R^2	DC	1.00	1.00
	PLT	0.96	0.94
	TR	0.74	0.90
Avg. Err. (%)	DC	0.00	0.00
	PLT	0.03	0.07
	TR	0.30	0.39

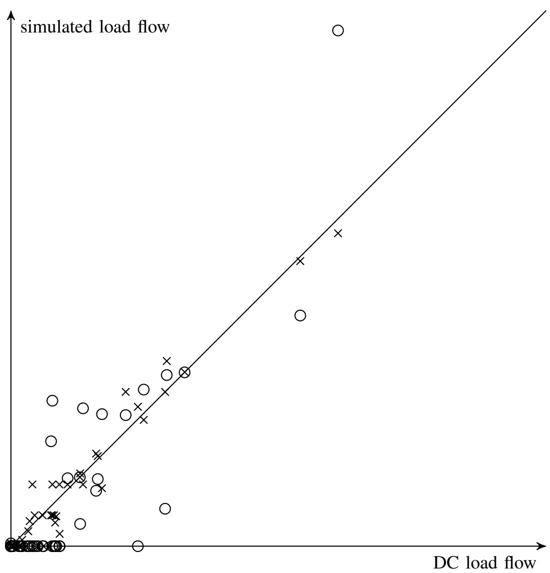
Table I
Accuracy Ratings

we assume a very simple unit dispatch model where p_r is only restricted by some fixed upper bound and κ_r is a linear (or convex piecewise linear) function. The methods that we have presented, however, make no use of this assumption and may therefore be used with any choice of unit dispatch/unit commitment model.

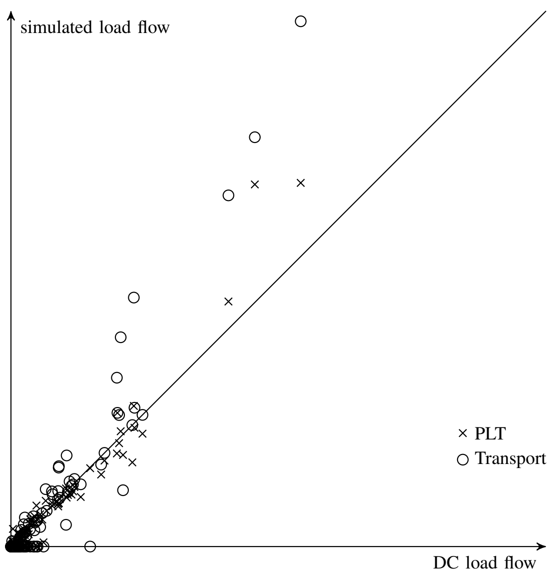
In total, 4 cases were tested with DC, PLT and TR load flow models, yielding 12 problems for which the results are presented below:

The accuracy ratings in Table I and the plots in Figure 3 demonstrate that the PLT model quite consistently beats the TR model in terms of mirroring the reference DC load flow. This advantage persists, regardless of whether the coefficient of determination R^2 or the average MW error relative to the line capacity is considered.

Looking at computations times in Table II, the qualitative disadvantage of the DC model compared to both other models is apparent: Transmission Capacity Expansion Problems using the PLT model may, just as for the Transport Model, be formulated as Linear Programs. Thus, computation times for both models are much shorter than for the DC Model, which requires non-polynomial-time solution techniques such as Mixed-Integer-Programming or Nonlinear Programming. It should be noted that only standard solution methods were



(a) DENA



(b) EU

Figure 3. This figure shows load flow values per line for the ED problem. Values both from the PLT and the Transport model are plotted against the reference values (from DC load flow), a dot on the diagonal represents a perfect fit.

applied to the Mixed-Integer problems. Thus, solution times might be better in practice, without, however, eliminating the qualitative difference between solution times for linear and mixed-integer problems. Computation times for the Economic Dispatch Problem are relatively fast in all load flow models.

IV. Conclusion

A. Conclusion

This paper introduced the PLT model and demonstrated the feasibility and advantages for solving TEP problems. It was shown that PLT's accuracy level is comparable to DC power flow. The PLT model contributes towards integrated energy systems optimization by offering a much more precise version of the Transport Model while maintaining its low computational complexity, even for the case of capacity expansion. This reduces complexity of the Transmission Capacity Expansion Planning problems to that of storage and generation capacity expansion, which can easily be solved even for large-scale problems.

	DENA		EU	
	ED	TEP	ED	TEP
DC	7.5	963	17.6	962.3
PLT	10.9	31.0	14.6	17.9
TR	5.6	13.6	14.5	15.8

Table II
Computation Times (s)

B. Future research

Findings from this paper suggest further research on (different aspects of) the PLT limitations: Most prominently the effects of the grid structure on the accuracy of load flow models and the integration of network reduction algorithms are subject to future investigation. Reactive power and voltage stability should be considered as a next step towards more accurate representation of the electricity grid.

It should be noted that DC load flow values do not coincide exactly with AC power flows. Still, the DC model is the most widely used (and best available) approximation of AC power flows in the domain of Mixed-Integer Linear Programming TEP problems. Hence, expansion decisions obtained from the TEP problem could be checked for feasibility by performing exact AC power flow calculations with line capacities fixed to the result values.

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