Technische Universität München
Zentrum Mathematik

Random Graphs (MA 5208) Summer 2012

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Sheet 2

Problem 2.1 *Quickly constructing a large network meeting given specifications.*
Suppose you need to construct (e.g. as a testing ground for some experiment) a graph G satisfying the specifications

(S.1) $|G| \geq 250$,

(S.2) $\|G\| > |G|^{1.001}$,

(S.3)  is not a subgraph of G .

You are willing to accept a chance of at most $p_{\text{tol}} := 10^{-3}$ that your construction does not satisfy the requirements. At your disposal you have a trustworthy Bernoulli(p)-source of randomness, for arbitrary p. What are you going to do? Prove that what you are planning to do meets the requirements. (Try to exercise some care to make economical choices for the parameters, in particular try to make your $|G| \geq 250$ not too large. However, this exercise does not ask you to find the smallest $|G|$ which works. Do not spend much time optimizing your estimates.)

Problem 2.2 *Almost always, all maximal cliques are large.*

(1) Let $s \in \mathbb{N}$. Find an example of a connected graph G which for every $2 \leq i \leq s$ contains an inclusion-maximal clique on i vertices.

(2) Prove that almost every graph G does not contain any inclusion-maximal clique with less than $\frac{1}{2}\log|G|$ vertices.

Problem 2.3 *Plane trees and binary trees.*
Find a bijection between plane trees with $n+1$ vertices and binary trees with n internal nodes.

Problem 2.4 *Streaks of luck and enumeration.*
Given a fixed integer $k \geq 2$, let a_n be the number of binary words of length n not containing k consecutive zeroes. Compute the generating function $\sum_{n\geq 0} a_n z^n$.

Problem 2.5 *Plane trees and ternary trees.*
Let T_n be the number of ternary trees with n internal nodes, and let E_n be the number of plane trees with n vertices such that every vertex has even outdegree. Find equations satisfied, respectively, by the generating functions

$$T(z) = \sum_{n\geq 0} T_n z^n \qquad \text{and} \qquad E(z) = \sum_{n\geq 0} E_n z^n \quad .$$

Deduce that $T_n = E_{2n+1}$ for all $n \geq 0$.