Technische Universität München
Zentrum Mathematik
Random Graphs (MA 5208) Summer 2012
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Sheet 6

Problem 6.1 *On chromatic number when the expected degree is larger than one.*
Prove that for every $\ell \in \mathbb{Z}_{\geq 3}$ there is $c_\ell > 1$ so that $p_n \geq \frac{c_\ell}{n}$ implies $\mathsf{Pr}_{\mathrm{G}(n,p_n)}\left[\chi(G) \leq \ell\right] \xrightarrow{n\to\infty} 0$.

Problem 6.2 *Chernoff-bounds, and concentration of the number of edges.*
Prove that if X is binom(n,p)-distributed, then for every $\delta \geq 0$,

$$\mathsf{Pr}[|X - \mathsf{Ex}[X]| \geq \delta] \leq \exp(-\tfrac{\delta^2}{2\cdot\mathsf{Ex}[X]}) \quad . \tag{1}$$

Now show that Chebyshev's inequality gives the estimate that for every *constant* $0 < p < 1$ there exists a constant $C_p > 0$ such that

$$\mathsf{Pr}_{\mathrm{G}(n,p)}\left[\Big|\|\cdot\| - \mathsf{Ex}[\|\cdot\|]\Big| > n^{3/2}\right] \leq \tfrac{C_p}{n} \quad . \tag{2}$$

whereas (1) tells us that for every constant $0 < p < 1$ there exists a constant $C_p > 0$ with

$$\mathsf{Pr}_{\mathrm{G}(n,p)}\left[\Big|\|\cdot\| - \mathsf{Ex}[\|\cdot\|]\Big| > n^{3/2}\right] \leq \exp(-\tfrac{n}{C_p}) \quad . \tag{3}$$

Problem 6.3 *An aspect of the phase transition for connectedness: isolated vertices.*
Prove that for every $c \in \mathbb{R}$,

$$\mathsf{Pr}_{\mathrm{G}(n,\frac{c+\log n}{n})}[\{G \in \mathcal{G}_n : \text{in } G \text{ there does not exist an isolated vertex}\}] \xrightarrow{n\to\infty} \exp(-\exp(-c)) \quad . \tag{4}$$