Technische Universität München
Zentrum Mathematik

Stochastic Programming

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Exercise sheet 2

Exercise 2.1 **The inventory model:** A company decides on the order quantity x to meet an uncertain demand d. The cost of the order is $c > 0$ per unit of demand x. If the demand is higher than the order, then an additional order can be made with unit costs $b > c$. On the other hand, if the demand is below the order quantity, then the surplus of x must be hold with costs $h \geq 0$.

1. Formulate the minimization of the total costs to order quantity x as a two-stage stochastic program.
2. Show that the objective function is a convex function.
3. Solve the problem by evaluating the expectation in the objective function. Use
$$\int_0^x P(d \leq z)dz = E_d(\max\{0, x - d\}).$$
4. Let $c = 0.5b$, $h = 9b$, $b > 0$ and let the demand d have the following distributions:
   a) d has an (continuous) uniform distribution on $[0, 20]$, i.e. $d \sim \mathrm{U}(0, 20)$.
   b) d has an exponential distribution with $\lambda = 0.1$, i.e. $d \sim \mathrm{Ex}(0.1)$.

   Let $\bar{d}$ be the corresponding mean value. Compute the expected value solution $\bar{x}$ and the optimal solution x^*. Compare both values and their objective value for the realisation of the demand $d = 8$.

Exercise 2.2 Consider the **inventory model**. We assume the company to be risk averse and introduce an upper bound on the total costs of τ for all realisations of the demand d (strict).

1. Formulate the problem of minimizing the total costs given the risk aversion.
2. For which realisations of the demand d does there exist **no** solution to the problem (for sure)?

Exercise 2.3 Consider again the **inventory model**. We assume the company to be risk averse and introduce an upper bound on the total costs of τ which is only allowed to be exceeded for at most $\alpha \in (0, 1)$ (significance level) of the realisations of the demand d.

Formulate the problem of minimizing the total costs given the risk aversion.