Stochastic Programming

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Exercise sheet 3

Exercise 3.1 Consider the **inventory model**. We assume the company to be risk averse and introduce an upper bound on the total costs of τ for all realisations of the demand d (strict).

1. Formulate the problem of minimizing the total costs given the risk aversion.
2. For which realisations of the demand d does there exist **no** solution to the problem (for sure)?

Exercise 3.2 Consider again the **inventory model**. We assume the company to be risk averse and introduce an upper bound on the total costs of τ which is only allowed to be exceeded for at most $\alpha \in (0, 1)$ (significance level) of the realisations of the demand d.

Formulate the problem of minimizing the total costs given the risk aversion.

Exercise 3.3 Let

$$\begin{aligned} \min\ & 2y_1 + y_2 \\ \text{s.t. } & y_1 + 2y_2 \geq \xi_1 - x_1, \\ & y_1 + y_2 \geq \xi_2 - x_1 - x_2, \\ & 0 \leq y_1 \leq 1, \\ & 0 \leq y_2 \leq 1 \end{aligned}$$

and $K_2(\xi) = \{x | x_1 \geq \xi_1 - 3, x_1 + x_2 \geq \xi_2 - 2\}$. Both random variables ξ_1 and ξ_2 follow a uniform distribution on $[2, 4]$. Compute K_2.

Exercise 3.4 **Benders' decomposition:** Consider the **facility location problem**: A company wants to expand into new territories and considers m possible locations for new factories, the cost of opening a factory at location i is f_i. A market analysis has revealed the demand for each of the n potential customers, the cost of supplying customer j from factory i is c_{ij}. We want to determine at which locations a factory should be opened and which fraction of each customer's demand should be supplied from which factory in order to minimize the total cost (delivery plus factory setup).

Use variables $x_{ij} \in [0; 1]$ to determine which fraction of customer j's demand should be supplied from the factory at location i and variables $y_i \in \{0, 1\}$ to determine whether or not to open a factory at location i.

1. Design an IP model for the facility location using the notation given above.

2. We want to apply Benders' decomposition to the problem. Formulate the dual subproblem for fixed y-variables. How can it be solved? Outline how to find a feasibility cut and an optimality cut.