Stochastic Programming

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Exercise sheet 4

Exercise 4.1, continued **Benders' decomposition:** Consider the **facility location problem**: A company wants to expand into new territories and considers m possible locations for new factories, the cost of opening a factory at location i is f_i. A market analysis has revealed the demand for each of the n potential customers, the cost of supplying customer j from factory i is c_{ij}. We want to determine at which locations a factory should be opened and which fraction of each customer's demand should be supplied from which factory in order to minimize the total cost (delivery plus factory setup).

Use variables $x_{ij} \in [0;1]$ to determine which fraction of customer j's demand should be supplied from the factory at location i and variables $y_i \in \{0,1\}$ to determine whether or not to open a factory at location i.

1. Design an IP model for the facility location using the notation given above.
2. We want to apply Benders' decomposition to the problem. Formulate the dual subproblem for fixed y-variables. How can it be solved? Outline how to find a feasibility cut and an optimality cut.

Exercise 4.2 **The L-Shaped Method**: Consider Step 3 of Iteration 1 within Example 1.

a) Let $\xi = \xi_1$, $x = x^1$ and

$$\begin{aligned} w = \min\ & -24y_1 - 28y_2 \\ & 6y_1 + 10y_2 \leq 2400 \\ & 8y_1 + 5y_2 \leq 1600 \\ & 0 \leq y_1 \leq 500 \\ & 0 \leq y_2 \leq 100. \end{aligned}$$

i) Check that the optimal dictionary (the simplex tableau in the optimal solution) is

$$\begin{aligned} w &= -6100 + 3s_2 + 13s_4 \\ s_1 &= 575 + 6/8s_2 + 50/8s_4 \\ y_1 &= 137.5 - 1/8s_2 + 5/8s_4 \\ s_3 &= 362.5 + 1/8s_2 - 5/8s_4 \\ y_2 &= 100 - s_4 \end{aligned}$$

for slack variables s_1, s_2, s_3, s_4.

ii) Check that this dictionary corresponds to the solution stated in Example 1.

iii) Check that the optimal value $w = -6100$ is also attained by the dual variables.

b) For $\xi = \xi_1$, the optimal solution is $w_1 = -6100$ and for $\xi = \xi_2$, the optimal solution is $w_2 = -8384$.

i) Check that $w^1 = 0.4w_1 + 0.6w_2$.

ii) Prove by linear programming duality that $w = \sum_{k=1}^{K} p_k w_k$, where w_k denotes the solution of the second-stage program, for realization k of ξ, $k = 1, \ldots, K$.

Exercise 4.3 Consider the following problem:

$$\begin{aligned} w = \min\ & 7x_1 + 11x_2 + E_\xi(q_1 y_1 + q_2 y_2) \\ & y_1 + 2y_2 \geq d_1 - x_1 \\ & y_1 \geq d_2 - x_2 \\ & 0 \leq x_1 \leq 10 \\ & 0 \leq x_2 \leq 10 \\ & y_1, y_2 \geq 0. \end{aligned}$$

where $\xi^\intercal = (q_1, q_2, d_1, d_2)$ takes on the values $(26, 16, 6, 12)$ and $(14, 24, 10, 4)$ with probability 0.5 each.

In this example, the L-Shaped method selects $x^1 = (0, 0)^\intercal$ as starting point. The L-Shaped method can however be used with any other reasonable starting point. Take $x^1 = (1, 5)^\intercal$ as starting point and show that exactly the same steps are taken if the starting point is any point within the region $4 \leq x_2 \leq 6 + x_1$.