Technische Universität München
Zentrum Mathematik

Stochastic Programming

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Exercise sheet 5

Exercise 5.1 Let z_{LR} be the optimal value of the Lagrange relaxation of $IP(Q)$: $\max\{c^\intercal x : A^1 <\leq b^1, x \in Q\}$, where $Q = \{x : A^2 x \leq b^2, x \geq 0\}$ and $z_{\text{IP}} = \max\{c^\intercal x : x \in S\}$, $S = \{x \in \mathbb{Z}^n : Ax \leq b, x \geq 0\}$.

$$z_{\text{LR}}(\lambda) := \max\{c^\intercal x + \lambda^\intercal (b^1 - A^1 x) : x \in Q\}.$$

Show that $z_{\text{LR}}(\lambda)$ is a convex function.

Exercise 5.2 Consider the following problem

$$\begin{array}{lll} \min & 3x_1 - x_2 & \\ \text{s.t.} & x_1 - x_2 & \geq -1 \\ & -x_1 + 2x_2 & \leq 5 \\ & 3x_1 + 2x_2 & \geq 3 \\ & 6x_1 + x_2 & \leq 15 \\ & x_1, x_2 & \geq 0 \\ & x_1, x_2 & \in \mathbb{Z}. \end{array}$$

Set $A^1 := \begin{pmatrix} -1 & 1 \end{pmatrix}$, $b^1 := 1$ and $A^2 := \begin{pmatrix} -1 & 2 \\ -3 & -2 \\ 6 & 1 \end{pmatrix}$, $b^2 := \begin{pmatrix} 5 \\ -3 \\ 15 \end{pmatrix}$. Compute the optimal value z_{IP} and the values z_{LD}, the Lagrange dual, and z_{LP}, the optimal solution of the linear relaxation.

Exercise 5.3

1. Find conditions on Q, S and $\{x : Ax \leq b\}$ such that (i) $z_{\text{IP}} = z_{\text{LD}}$, for all objective vectors c, and (ii) $z_{\text{LP}} = z_{\text{LD}}$, for all objective vectors c.
2. Consider the problem of Exercise 4.2. Find examples of objective vectors c such that (i) $z_{\text{LP}} < z_{\text{LD}} < z_{\text{IP}}$, (ii) $z_{\text{LP}} < z_{\text{LD}} = z_{\text{IP}}$ and (iii) $z_{\text{LP}} = z_{\text{LD}} = z_{\text{IP}}$.

Exercise 5.4 Consider again the problem of Exercise 4.2. Solve this problem using the subgradient approach applied to the Lagrange dual (for at least 10 iterations). For each iteration k use a step length of $s_k = 0.8^k$.