Stochastic Programming

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Exercise sheet 6

Exercise 6.1 Let the following two versions of the Gordon-Dickson-Lemma be given.

Sequence version:
Let $\{p_1, p_2, ...\}$ be a sequence of points in $\mathbb{Z}_+^n$ such that $p_i \not\leq p_j$ whenever $i < j$. This sequence is finite.

Set version:
Every infinite set $S \subseteq \mathbb{Z}_+^n$ contains only finitely many $\leq -$ minimal points.

Show that both versions of the Gordon-Dickson-Lemma are equivalent.

Exercise 6.2 Proof the Sequence version of the Gordon-Dickson-Lemma by induction on the dimension n.

Exercise 6.3 Let z be a feasible solution to $\min\{f(z) : Az = b, l \leq z \leq u, z \in \mathbb{Z}^n\}$ and let $z^\star$ be an optimal solution of it, $z \neq z^\star$. Show that there exists some $\alpha_i \in \mathbb{R}_+$ and $g_i \in \mathcal{G}(A)$ such that $z + \alpha_i g_i$ is again a feasible solution to the integer program above.

Exercise 6.4 For any given matrix $A \in \mathbb{Z}^{m\times n}$ and for every orthant $\mathbb{O}_j$ of $\mathbb{R}^n$, let $\mathcal{H}_j$ denote the unique inclusion-minimal Hilbert basis of the pointed rational polyhedral cone $\mathbb{O}_j \cap ker_{\mathbb{Z}^n}(A)$. Show, that the set $\mathcal{G}_1(A) := \{g \in ker_{\mathbb{Z}^n}(A) \setminus \{0\} : \nexists h \in \mathcal{G}_1(A), h \sqsubseteq g\}$ is equal to the Graver basis $\mathcal{G}_2(A) := \bigcup_{j=1}^{2^n} \mathcal{H}_j \setminus \{0\}$.