Technische Universität München, Zentrum Mathematik
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Discrete Optimization (MA 3502)

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Exercise Sheet 4

Exercise 4.1
Consider the linear system of equations $Ax = b$ with

$$A = \begin{pmatrix} 4 & 2 & 2 & 2 \\ 1 & 3 & 4 & 5 \end{pmatrix}, b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$

Determine all vectors $b \in \mathbb{Z}^2$ for which there exists an integer solution $x \in \mathbb{Z}^4$ of this system, and determine for each such b the set of solutions $L(b) := \{x \in \mathbb{Z}^4 : Ax = b\}$.

Exercise 4.2
Let P and Q be polytopes in $\mathbb{R}^n$ where $n \geq 1$. Prove or disprove: $I(P \setminus Q) = I(P) \setminus I(Q)$.

Exercise 4.3
Let $P \subset \mathbb{R}^2$ be defined through the following $\mathcal{H}$-presentation:

$$-\sqrt{2}x_1 + x_2 \leq 0$$
$$x_1 - \sqrt{2}x_2 \leq 0$$

a) Sketch both P and the integral points contained in P. Take a guess at the integer hull $I(P) := \text{conv}(P \cap \mathbb{Z}^2)$ based on your sketch!

b) Show that $P = \text{pos}\left\{(1, \sqrt{2})^T, (\sqrt{2}, 1)^T\right\}$ ($\mathcal{V}$-presentation).

c) Prove that $I(P) = \{0\} \cup \text{int}\,(P)$. You may use the following result without proof: For every line in $\mathbb{R}^2$ with irrational slope there exist integer points arbitrarily close to (and on both sides of) the line.

d) Is $I(P)$ a polyhedron?

e) Let $Q := \left\{(-\frac{1}{2}, -\frac{1}{2})^T\right\} + P$. Show that the integer hull $I(Q)$ of Q has an infinite number of vertices.