1: Find the Smith Normal Form of the following matrices

(a)

$$\begin{pmatrix} 4 & 2 & -1 \\ 3 & 1 & 0 \\ 0 & 7 & 2 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 7 & 0 \\ 0 & 2 \end{pmatrix}$$

via operations on columns and rows that are invertible on $\mathbb{Z}^{n\times n}$. You should track the operations you perform and collect them in two matrices $U, V \in Gl_n(\mathbb{Z})$ such that $UAV = D$, where $D \in \mathbb{Z}^{n\times n}$ is a diagonal matrix, whose entries on the diagonal $d_1, \ldots, d_n$ are such that $d_1|d_2|\ldots|d_n$.
This form is unique up to multiplication with ± 1. Moreover, one can show that

$$d_1 \cdots d_i = g.c.d\{A_j \mid A_j \text{ is an } i \times i \text{ submatrix of } A, \forall j\}$$

Remark: a slightly weaker factorization is just as useful for solving binomial systems: just reduce to a diagonal matrix (and neglect the divisibility condition). You lose uniqueness, but the resulting normal form is equally useful for solving the system (and quicker to get!).

2:

(a) Find all the third-powers in $\mathbb{F}_{27}$.

(b) How many solutions does $x^4 = 6$ have in $\mathbb{F}_{17}$?

Remark: Recall exercise 5 of hw1:

$$x^d = c \text{ has a root in } \mathbb{F}_q \Leftrightarrow c^{\frac{q-1}{gcd(q-1,d)}} = 1 \text{ in } \mathbb{F}_q \qquad q = p^k$$