# Problem Set 7

**Exercise 7.1** (Christofides heuristic)
The Christofides heuristic approximates METRIC TSP within a factor of 3/2 (see lectures). Give an example that shows that this bound cannot be improved, i. e., construct instances of METRIC TSP (for general vertex numbers) such that Christofides yields a solution that asymptotically achieves this approximation ratio.

**Exercise 7.2** (Approximating METRIC HAMILTONIAN PATH)
Consider variants of the METRIC TSP problem in which the object is to find a (simple) path containing all the vertices of the graph. Three different problems arise, depending on the number (0, 1, or 2) of endpoints of the path that are specified. By modifying the Christofides heuristic obtain a 3/2 factor approximation algorithm for the case that zero or one endpoint is specified.

**Exercise 7.3** (Approximating a special case of the TSP)
Let $G = (V, E)$ be a complete undirected graph where the edge lengths $c(e)$ for every $e \in E$ are elements of $\{1, 2\}$. This graph satisfies clearly the triangle inequality.

(a) Give a 2 factor approximation algorithm for TSP in this special class of graphs.

(b) Give a 4/3 factor approximation algorithm for TSP in this special class of graphs.

*Hint:* Start by finding a minimum 2-matching in $G$, then patch cycles together. (A 2-matching is a subset $S$ of edges so that every vertex has exactly 2 edges of $S$ incident at it.)

**Exercise 7.4** (Dynamic Programming)
Let $c = (22, 64, 48, 100)^T$, $a = (2, 4, 3, 4)^T$ and $\beta = 6$ define a 0-1-knapsack problem. Solve this knapsack problem by dynamic programming.