

Subsection 1

Separating Odd Cycle Inequalities for STABLE SET

Recap: STABLE SET

ILP / LP:

$$\begin{aligned}x_i + x_j &\leq 1 \quad \forall \{i, j\} \in E \\x &\in \{0, 1\}^n\end{aligned}$$

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Odd Set Inequalities:

$$\sum_{v \in K} x_v \leq \frac{|K| - 1}{2}$$

Odd Cycle Separation Problem

Input: STABLE SET instance, feasible LP solution x^*

Task: Find violated odd cycle inequality:

$$\sum_{v \in K} x_v^* > \frac{|K| - 1}{2}$$

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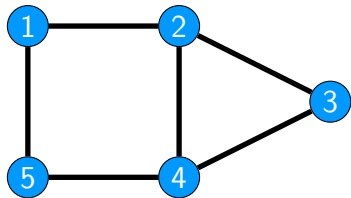
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Lemma

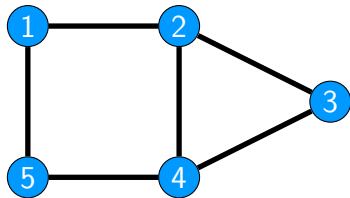
Set $y_{ij} := 1 - x_i^* - x_j^*$

$$\rightarrow \sum_{v \in K} x_v^* > \frac{|K| - 1}{2} \Leftrightarrow \sum_{\{i,j\} \in E(K)} y_{ij} < 1$$

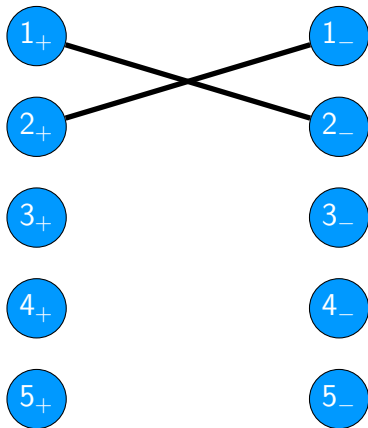
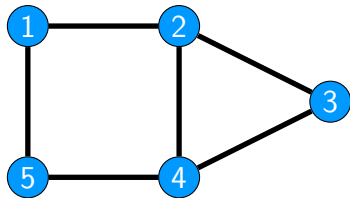
Graph Transformation for Odd Cycle Separation



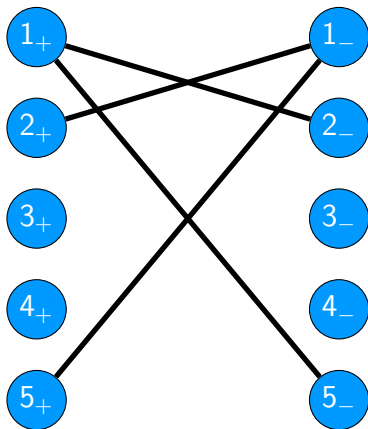
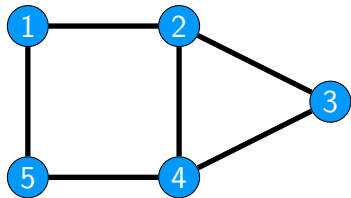
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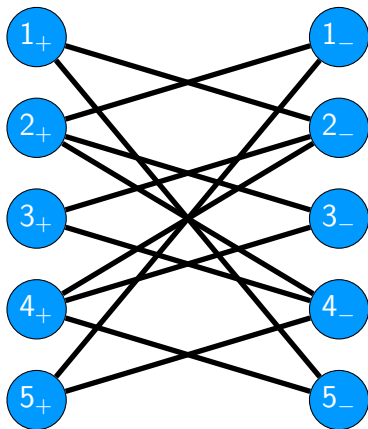
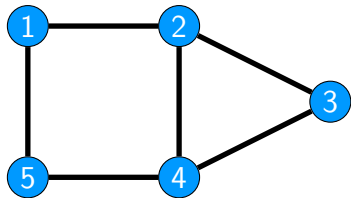
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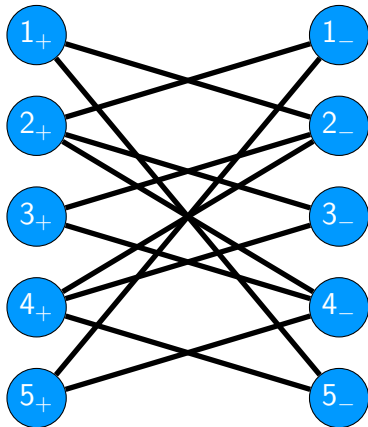
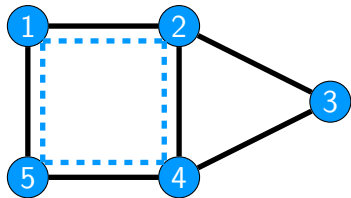
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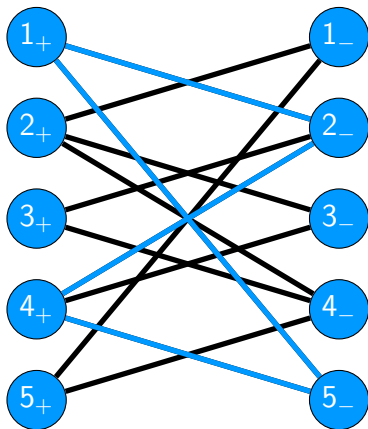
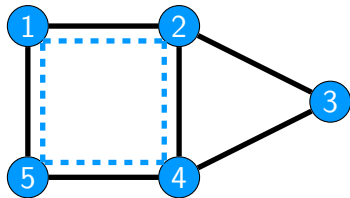
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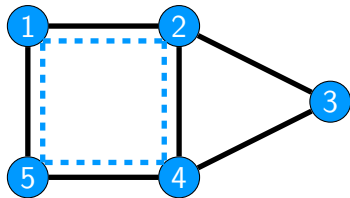
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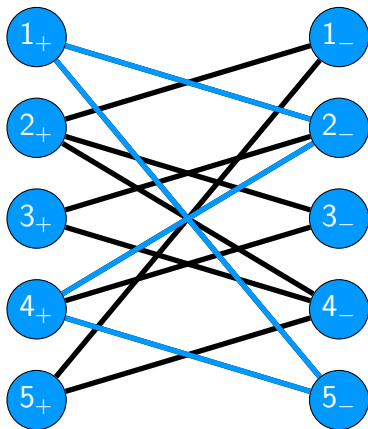


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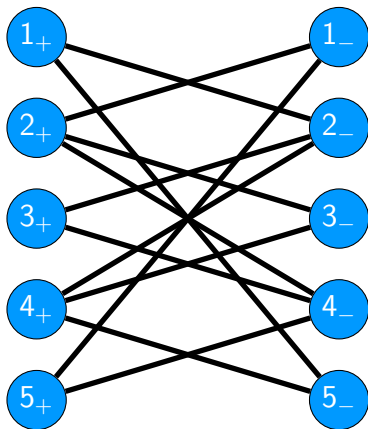
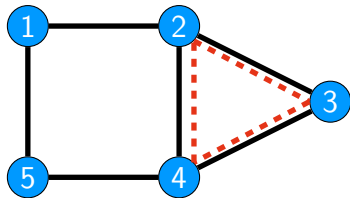


even cycle

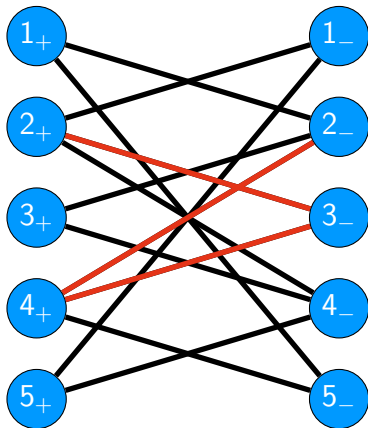
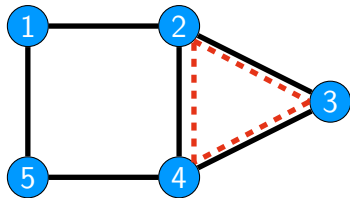
→ even $i_+ - i_+$ cycle



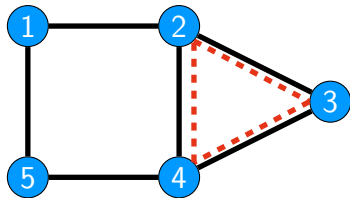
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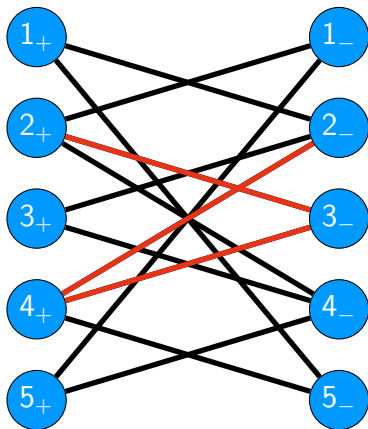


Graph Transformation for Odd Cycle Separation



odd cycle

→ odd i_+ - i_- path



Separation Procedure

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4. violated inequalities found \rightarrow start over