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Network flows
Summer term 2016

Problem set 4

Discussion: June 23, 2016

Exercise 4.1
Consider a transshipment problem with a given vector $b \in \mathbb{R}^V$ of supplies ($b(v) > 0$) and demands ($b(v) < 0$) at the nodes of the network G. We have seen in the lecture that the problem of finding a feasible b-flow that satisfies all supplies and demands can be reduced to a maximum s-t-flow problem by introducing a super source s and a super sink t. Try to find a similar reduction for the case of flows over time and discuss the principal difficulties that occur.

Exercise 4.2
Find a network such that no temporally repeated flow has the earliest arrival property.

Exercise 4.3
The definition of earliest arrival flows can be generalized to a setting with multiple sources and sinks, each with a given supply. It turns out that an earliest arrival flow still exists in this case. On the other hand, we run into problems when there are multiple sinks $t_1, \dots, t_k$ with given demands $d_1, \dots, d_k$ that bound the excess of the sinks. That is, $\mathrm{ex}(x, t_i, \theta) \leq d_i$ must hold for all i and θ. Construct an example with two sinks and one source for which no flow over time x maximizes $\mathrm{ex}(x, t_1, \theta) + \mathrm{ex}(x, t_2, \theta)$ for all θ simultaneously.

Exercise 4.4
We consider temporally repeated flows in the context of minimum cost flows over time.

a) Construct a network with two temporally repeated flows x and x' corresponding to two different decompositions of the same static s-t-flow x such that $\mathrm{cost}(x) \neq \mathrm{cost}(x')$.

b) Construct an example in which the cost of any temporally repeated flow with time horizon T and value d is larger than the minimum cost of an s-t-flow over time with time horizon T and value d.

Exercise 4.5
Show that an arbitrary s_1-t_T-cut C of finite capacity in the time-expanded network D_T naturally induces an s-t-cut over time with time horizon T and $\mathrm{cap}(C) = \mathrm{cap}(\alpha)$ in D.
The concept of time-expanded networks will be introduced in the upcoming lecture on June 21.