Proof of Lemma 2.3:

"$\Rightarrow$": Assume d is conservative.

Define $D^* = (V \cup \{s\}, A \cup \{(s,v) : v \in V\})$ and $d(s,v) := 0 \ \forall v \in V$.

Then d is conservative in D^* as well and MBF computes $\phi : V \to \mathbb{R}$ such that $\phi(v) = \min_{P:\, P\, s\text{-}v\text{-path}} d(P)$.

Let $(u,v) \in A$. Let P^* s-u-path with $d(P^*) = \phi(u)$.

Then $P^* \cup \{(u,v)\}$ is s-v-path and thus $\phi(v) \leq d(P^*) + d(u,v)$.

$\Rightarrow$ ϕ is feasible potential.

"$\Leftarrow$": Assume ϕ is feasible potential. Let C be a cycle in D.

Then $\sum_{(u,v) \in C} d(u,v) \geq \sum_{(u,v) \in A} \phi(v) - \phi(u) = 0$. $\Rightarrow$ d is conservative. $\square$

Proof of Lemma 2.4:

- $d'(u,v) \geq 0$ follows immediately from definition of feasible potentials.
- $d'(P) = \sum_{(u',v') \in P} d(u',v') + \phi(u') - \phi(v') = d(P) + \phi(v) - \phi(w)$ $\square$

Application: Compute shortest path for every pair $s,t \in V$.
(see Thm. 7.8 in KV)